Low-temperature expansion data for the Ising model: simple cubic and simple quadratic lattices

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COMMENT

Low-temperature expansion data for the Ising model: simple cubic and simple quadratic lattices

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Abstract. Two new high-field polynomials L_{14} and L_{15} are given for the simple cubic lattice together with the complete partial generating function F_7 which determines the corresponding sub-lattice polynomials. For the simple quadratic lattice the complete partial generating function F_8 is given; this, combined with the recent work of Baxter and Enting, determines the corresponding sub-lattice polynomials to order 19. Extra high-field polynomials through L_{21} derived by manipulation of the results of Baxter and Enting are quoted explicitly.

In this Comment we report new data for series expansions for the Ising model on the simple cubic and simple quadratic lattices. Recently Sykes (1979) has derived complete partial generating functions (codes) F_0–F_7 for the four-dimensional simple cubic lattice and these provide sufficient information to determine all the high-field sub-lattice polynomials up to 15th order in the field variables μ and ν. Using essentially the same techniques we have carried the corresponding calculations for the simple cubic lattice to the same order and have extended the work of Sykes *et al* (1973a) by providing the complete partial generating function F_7. We give this and the high-field polynomials L_{14} and L_{15} derived from it in appendix 1.

For the simple quadratic lattice we have extended the work of Sykes *et al* (1973b) by adding the complete partial generating function F_8. We give this in appendix 2 together with the high-field polynomials L_{16} and L_{17} derived therefrom. These latter are in agreement with the recent results of Baxter and Enting (1978) who have derived the expansion of the partition function of this lattice as a temperature grouping through u^{23}. Their results implicitly determine the high-field polynomials through L_{23}; since the rearrangement is tedious we quote these useful polynomials explicitly through L_{21} in appendix 2. Beyond this the size of the intermediate numbers has prevented our completing the manipulation with our existing computer routines but we hope to provide the remainder subsequently. The polynomials L_{18} and L_{19} combined with the data of F_8 determine the corresponding sub-lattice polynomials up to 19th order. We take this opportunity of recording that the last two coefficients for the high-temperature susceptibility of the simple quadratic lattice given by Sykes *et al* (1972) (equation (3.1)) should be corrected to the values $377\,009\,364v^{20}$ and $942\,106\,116v^{21}$ in agreement with the results of Baxter and Enting (1978).

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Appendix 1

Complete codes F_n and high-field polynomials L_n for the simple cubic lattice.

$$\begin{aligned}F_7 = {} & 8(22, 9, 9, 3, 0, 0, 1)+24(23, 9, 12, 1, 0, 0, 1)+6(24, 10, 13, 0, 0, 0, 1)\\
& +60(24, 11, 10, 2, 0, 1)+48(24, 11, 11, 0, 1, 1)+4(24, 12, 6, 6)\\
& +12(24, 12, 8, 2, 2)+24(24, 12, 8, 3, 0, 1)+72(24, 13, 6, 4, 0, 1)\\
& +168(24, 13, 7, 2, 1, 1)+12(24, 15, 2, 6, 0, 1)+24(24, 15, 3, 4, 1, 1)\\
& +120(25, 12, 11, 1, 0, 1)-44(25, 12, 12, 0, 0, 0, 1)+120(25, 13, 7, 5)\\
& +384(25, 13, 8, 3, 1)+264(25, 13, 9, 1, 2)+432(25, 13, 9, 2, 0, 1)\\
& +48(25, 13, 10, 0, 1, 1)+144(25, 14, 6, 4, 1)+120(25, 14, 7, 2, 2)\\
& +240(25, 14, 7, 3, 0, 1)+120(25, 15, 4, 5, 1)+432(25, 15, 5, 3, 2)\\
& +240(25, 15, 5, 4, 0, 1)+144(25, 15, 6, 1, 3)+48(25, 16, 3, 4, 2)\\
& +72(25, 17, 2, 3, 3)+72(26, 13, 11, 1, 1)+180(26, 13, 12, 0, 0, 1)\\
& +660(26, 14, 8, 4)+1\,680(26, 14, 9, 2, 1)+444(26, 14, 10, 0, 2)\\
& +1\,080(26, 14, 10, 1, 0, 1)+744(26, 15, 6, 5)+2\,640(26, 15, 7, 3, 1)\\
& +1\,152(26, 15, 8, 1, 2)+684(26, 15, 8, 2, 0, 1)+168(26, 16, 4, 6)\\
& +1\,584(26, 16, 5, 4, 1)+1\,308(26, 16, 6, 2, 2)+324(26, 16, 6, 3, 0, 1)\\
& +360(26, 17, 3, 5, 1)+732(26, 17, 4, 3, 2)+4(26, 18, 0, 8)\\
& +24(26, 18, 1, 6, 1)+108(26, 18, 2, 4, 2)+48(27, 13, 13, 1)\\
& +144(27, 14, 12, 0, 1)+4\,032(27, 15, 9, 3)+5\,016(27, 15, 10, 1, 1)\\
& +660(27, 15, 11, 0, 0, 1)+6\,792(27, 16, 7, 4)+12\,984(27, 16, 8, 2, 1)\\
& +1\,128(27, 16, 9, 0, 2)-24(27, 16, 9, 1, 0, 1)+3\,240(27, 17, 5, 5)\\
& +9\,336(27, 17, 6, 3, 1)+744(27, 17, 7, 1, 2)+552(27, 18, 3, 6)\\
& +3\,336(27, 18, 4, 4, 1)+1\,416(27, 18, 5, 2, 2)-1\,176(27, 18, 5, 3, 0, 1)\\
& +216(27, 19, 2, 5, 1)+96(27, 20, 1, 4, 2)+24(28, 14, 14)\\
& +128(28, 15, 12, 1)+17\,604(28, 16, 10, 2)+6\,264(28, 16, 11, 0, 1)\\
& +39\,300(28, 17, 8, 3)+30\,000(28, 17, 9, 1, 1)\\
& -2\,934(28, 17, 10, 0, 0, 1)+22\,948(28, 18, 6, 4)\\
& +18\,048(28, 18, 7, 2, 1)-1\,602(28, 18, 8, 0, 2)\\
& -4\,992(28, 18, 8, 1, 0, 1)+5\,880(28, 19, 4, 5)+9\,912(28, 19, 5, 3, 1)\\
& +48(28, 19, 6, 1, 2)+240(28, 20, 2, 6)-792(28, 20, 3, 4, 1)\\
& -4\,974(28, 20, 4, 2, 2)-312(28, 22, 0, 4, 2)+216(29, 16, 13)\\
& +40\,992(29, 17, 11, 1)+136\,344(29, 18, 9, 2)+25\,080(29, 18, 10, 0, 1)\end{aligned}$$

$+90\,636(29,19,7,3)+1\,668(29,19,8,1,1)-4\,164(29,19,9,0,0,1)$
$+21\,312(29,20,5,4)-24\,792(29,20,6,2,1)-3\,144(29,20,7,0,2)$
$-1\,896(29,21,3,5)-24\,816(29,21,4,3,1)-5\,232(29,22,2,4,1)$
$+38\,310(30,18,12)+279\,468(30,19,10,1)+171\,684(30,20,8,2)$
$-73\,416(30,20,9,0,1)-91\,004(30,21,6,3)-186\,900(30,21,7,1,1)$
$+9\,000(30,21,8,0,0,1)-56\,508(30,22,4,4)-45\,264(30,22,5,2,1)$
$-336(30,22,6,0,2)-6\,912(30,23,2,5)-24\,324(30,23,3,3,1)$
$+270\,816(31,20,11)-89\,436(31,21,9,1)-960\,552(31,22,7,2)$
$-251\,532(31,22,8,0,1)-396\,036(31,23,5,3)$
$-93\,276(31,23,6,1,1)-88\,580(31,24,3,4)-5\,040(31,24,4,2,1)$
$+39\,432(31,25,2,3,1)-440\,622(32,22,10)-3\,486\,528(32,23,8,1)$
$-1\,853\,494(32,24,6,2)+301\,824(32,24,7,0,1)-92\,616(32,25,4,3)$
$+237\,480(32,25,5,1,1)+84\,753(32,26,2,4)-5\,340\,540(33,24,9)$
$-2\,476\,404(33,25,7,1)+1\,859\,136(33,26,5,2)$
$+517\,740(33,26,6,0,1)+289\,116(33,27,3,3)$
$+47\,400(33,27,4,1,1)+75\,180(33,28,1,4)+773\,346(34,26,8)$
$+14\,219\,460(34,27,6,1)+3\,484\,998(34,28,4,2)$
$-393\,168\,(34,28,5,0,1)+80\,676(34,29,2,3)-79\,708(34,30,0,4)$
$+37\,076\,556(35,28,7)+9\,914\,988(35,29,5,1)$
$-1\,147\,776(35,30,3,2)-141\,480(35,30,4,0,1)-695\,967(36,30,6)$
$-23\,675\,928(36,31,4,1)-1\,363\,800(36,32,2,2)$
$-115\,397\,940(37,32,5)-9\,098\,592(37,33,3,1)$
$+20\,212\,854(38,34,4)+13\,960\,308(38,35,2,1)$
$+160\,868\,540(39,36,3)-58\,912\,425(40,38,2)$
$-80\,454\,540(41,40,1)+42\,154\,751\tfrac{1}{7}(42,42)$

$L_{14}=114u^{19}+2\,340u^{20}+27\,100u^{21}+376\,368u^{22}+1\,910\,556u^{23}$
$+12\,211\,288u^{24}+38\,655\,507u^{25}-111\,028\,314u^{26}-2\,459\,002\,958u^{27}$
$-4\,759\,009\,662u^{28}+31\,337\,777\,631u^{29}+336\,840\,884\,287u^{30}$
$-1\,116\,282\,652\,284u^{31}-8\,226\,496\,562\,196u^{32}$
$+68\,881\,358\,304\,579u^{33}-242\,877\,501\,941\,709u^{34}$
$+530\,473\,248\,654\,447\tfrac{3}{7}u^{35}-793\,795\,235\,175\,230u^{36}$
$+843\,636\,836\,811\,618u^{37}-640\,466\,975\,067\,133\tfrac{1}{2}u^{38}$
$+341\,119\,901\,759\,960u^{39}-121\,485\,656\,918\,688u^{40}$

$+26\,040\,745\,341\,858u^{41}-2\,544\,845\,359\,479u^{42}$

$$L_{15}=168u^{20}+4\,180u^{21}+54\,600u^{22}+547\,191u^{23}+4\,637\,272u^{24}$$
$$+17\,707\,032u^{25}+91\,207\,824u^{26}+31\,106\,906u^{27}-2\,625\,397\,110u^{28}$$
$$-20\,839\,441\,815u^{29}+12\,225\,153\,468u^{30}+438\,010\,067\,337u^{31}$$
$$+1\,919\,435\,831\,820u^{32}-16\,108\,095\,864\,028u^{33}$$
$$-31\,887\,595\,270\,824u^{34}+598\,432\,008\,384\,696u^{35}$$
$$-2\,711\,704\,283\,996\,334\tfrac{2}{3}u^{36}+7\,167\,431\,770\,407\,414u^{37}$$
$$-12\,843\,317\,422\,019\,148u^{38}+16\,468\,582\,458\,498\,264u^{39}$$
$$-15\,396\,815\,956\,875\,895\tfrac{1}{5}u^{40}+10\,467\,738\,098\,539\,380u^{41}$$
$$-5\,058\,600\,673\,985\,632u^{42}+1\,651\,212\,725\,909\,661u^{43}$$
$$-327\,036\,708\,321\,246u^{44}+29\,727\,323\,114\,819\tfrac{14}{15}u^{45}.$$

Appendix 2

Complete codes F_n and high-field polynomials L_n for the simple quadratic lattice.

$$F_8=2(15,4,8,0,3)+4(15,5,6,1,3)+1(16,4,8,4)+16(16,5,8,1,2)$$
$$+68(16,6,6,2,2)+48(16,7,4,3,2)+2(16,8,2,4,2)$$
$$+8(17,5,10,1,1)+32(17,6,7,4)+116(17,6,8,2,1)$$
$$+18(17,6,9,0,2)+8(17,7,5,5)+308(17,7,6,3,1)+80(17,7,7,1,2)$$
$$+238(17,8,4,4,1)+36(17,8,5,2,2)+28(17,9,2,5,1)$$
$$+2(17,10,0,6,1)+2(18,4,14)+24(18,5,12,1)+176(18,6,10,2)$$
$$+8(18,6,11,0,1)+580(18,7,8,3)+256(18,7,9,1,1)$$
$$+842(18,8,6,4)+976(18,8,7,2,1)-74\tfrac{1}{2}(18,8,8,0,2)$$
$$+444(18,9,4,5)+612(18,9,5,3,1)-368(18,9,6,1,2)$$
$$+72(18,10,2,6)+128(18,10,3,4,1)-138(18,10,4,2,2)$$
$$+48(19,6,13)+524(19,7,11,1)+2\,514(19,8,9,2)$$
$$+92(19,8,10,0,1)+4\,108(19,9,7,3)-144(19,9,8,1,1)$$
$$+2\,380(19,10,5,4)-3\,428(19,10,6,2,1)-312(19,10,7,0,2)$$
$$+344(19,11,3,5)-2\,076(19,11,4,3,1)-400(19,12,2,4,1)$$
$$+297(20,8,12)+1\,684(20,9,10,1)-762(20,10,8,2)$$
$$-944(20,10,9,0,1)-10\,756(20,11,6,3)-9\,056(20,11,7,1,1)$$
$$-7\,710(20,12,4,4)-6\,120(20,12,5,2,1)+630(20,12,6,0,2)$$
$$-1\,176(20,13,2,5)-968(20,13,3,3,1)-2\,356(21,10,11)$$
$$-25\,220(21,11,9,1)-72\,334(21,12,7,2)-2\,399(21,12,8,0,1)$$

$-46\,824(21,13,5,3)+9\,608(21,13,6,1,1)-7\,832(21,14,3,4)$
$+10\,276(21,14,4,2,1)+1\,464(21,15,2,3,1)-24\,960(22,12,10)$
$-80\,148(22,13,8,1)+6\,592(22,14,6,2)+17\,218(22,14,7,0,1)$
$+61\,280(22,15,4,3)+31\,408(22,15,5,1,1)+9\,962(22,16,2,4)$
$+29\,106(23,14,9)+376\,196(23,15,7,1)+454\,424(23,16,5,2)$
$-670(23,16,6,0,1)+64\,092(23,17,3,3)-33\,216(23,17,4,1,1)$
$+5\,700(23,18,1,4)+463\,293(24,16,8)+557\,052(24,17,6,1)$
$-232\,590(24,18,4,2)-53\,446(24,18,5,0,1)-60\,712(24,19,2,3)$
$-5\,027(24,20,0,4)-316\,472(25,18,7)-2\,096\,520(25,19,5,1)$
$-594\,792(25,20,3,2)+40\,127(25,20,4,0,1)-3\,136\,058(26,20,6)$
$-327\,368(26,21,4,1)+410\,768(26,22,2,2)+2\,732\,604(27,22,5)$
$+3\,481\,432(27,23,3,1)+8\,067\,417(28,24,4)-1\,863\,456(28,25,2,1)$
$-9\,677\,576(29,26,3)-5\,113\,942(30,28,2)+10\,639\,512(31,30,1)$
$-3\,668\,936\tfrac{5}{8}(32,32)$

$L_{16}=u^8+218u^9+4\,566u^{10}+30\,892u^{11}+3\,623u^{12}-1\,108\,184u^{13}$
$-7\,740\,717u^{14}+7\,558\,732u^{15}+252\,328\,534\tfrac{1}{4}u^{16}+695\,663\,502u^{17}$
$-7\,763\,969\,109u^{18}-14\,041\,788\,974u^{19}+153\,200\,490\,618\tfrac{1}{2}u^{20}$
$+411\,958\,890\,946u^{21}-6\,826\,079\,819\,049u^{22}+31\,473\,820\,047\,462u^{23}$
$-86\,032\,374\,205\,115\tfrac{1}{4}u^{24}+160\,585\,150\,845\,884u^{25}$
$-215\,410\,442\,159\,335u^{26}+211\,254\,152\,689\,798u^{27}$
$-150\,920\,915\,130\,185\tfrac{1}{2}u^{28}+76\,721\,425\,465\,850u^{29}$
$-26\,359\,477\,190\,004u^{30}+5\,496\,427\,483\,874u^{31}$
$-525\,988\,393\,828\tfrac{1}{16}u^{32}$

$L_{17}=88u^9+2\,978u^{10}+28\,744u^{11}+82\,996u^{12}-643\,196u^{13}$
$-7\,044\,777u^{14}-22\,662\,388u^{15}+176\,214\,842u^{16}+1\,213\,125\,160u^{17}$
$-902\,493\,482u^{18}-44\,794\,847\,116u^{19}+43\,016\,179\,894u^{20}$
$+860\,460\,546\,576u^{21}-651\,068\,372\,799u^{22}-28\,609\,686\,534\,092u^{23}$
$+183\,231\,125\,797\,020u^{24}-617\,342\,077\,087\,788u^{25}$
$+1\,382\,410\,263\,712\,281u^{26}-2\,218\,939\,502\,547\,292u^{27}$
$+2\,630\,877\,669\,812\,174u^{28}-2\,322\,036\,254\,921\,788u^{29}$
$+1\,512\,418\,551\,499\,516u^{30}-707\,925\,141\,207\,072u^{31}$
$+225\,715\,911\,311\,210u^{32}-43\,959\,196\,551\,212u^{33}$
$+3\,950\,266\,599\,523\tfrac{1}{17}u^{34}$

$$L_{18}=30u^9+1\,728u^{10}+23\,910u^{11}+116\,409\tfrac{1}{2}u^{12}-142\,716u^{13}$$
$$-5\,813\,746u^{14}-31\,382\,375\tfrac{1}{3}u^{15}+10\,361\,818u^{16}+1\,346\,305\,894u^{17}$$
$$+3\,498\,010\,849\tfrac{1}{3}u^{18}-25\,281\,047\,722u^{19}-186\,998\,502\,890u^{20}$$
$$+799\,737\,752\,060u^{21}+3\,209\,675\,885\,214u^{22}-15\,220\,831\,594\,346u^{23}$$
$$-91\,375\,786\,824\,327\tfrac{1}{3}u^{24}+960\,181\,669\,621\,346u^{25}$$
$$-4\,071\,360\,333\,943\,214u^{26}+10\,931\,535\,139\,041\,263\tfrac{5}{9}u^{27}$$
$$-20\,794\,259\,324\,497\,891\tfrac{1}{2}u^{28}+29\,304\,381\,108\,609\,908u^{29}$$
$$-31\,139\,167\,909\,740\,044u^{30}+24\,986\,674\,084\,840\,448u^{31}$$
$$-14\,954\,600\,950\,348\,898u^{32}+6\,485\,935\,826\,475\,579\tfrac{1}{3}u^{33}$$
$$-1\,929\,115\,566\,850\,030u^{34}+352\,442\,561\,182\,566u^{35}$$
$$-29\,851\,637\,540\,823\tfrac{11}{18}u^{36}$$

$$L_{19}=8u^9+914u^{10}+17\,716u^{11}+126\,862u^{12}+181\,600u^{13}-3\,565\,955u^{14}$$
$$-34\,318\,400u^{15}-80\,256\,628u^{16}+702\,818\,048u^{17}+7\,028\,630\,234u^{18}$$
$$-3\,646\,790\,564u^{19}-188\,701\,667\,538u^{20}-449\,418\,897\,300u^{21}$$
$$+6\,097\,882\,262\,514u^{22}+3\,538\,049\,680\,172u^{23}$$
$$-110\,879\,049\,045\,802u^{24}-115\,184\,404\,845\,400u^{25}$$
$$+4\,417\,024\,101\,978\,609u^{26}-24\,633\,399\,388\,928\,784u^{27}$$
$$+79\,817\,667\,961\,783\,426u^{28}-179\,098\,119\,452\,539\,004u^{29}$$
$$+296\,612\,206\,632\,305\,086u^{30}-372\,754\,826\,425\,435\,356u^{31}$$
$$+358\,923\,770\,106\,712\,494u^{32}-264\,034\,480\,122\,358\,200u^{33}$$
$$+146\,170\,993\,897\,152\,163u^{34}-59\,059\,298\,161\,782\,544u^{35}$$
$$+16\,460\,664\,578\,350\,944u^{36}-2\,831\,978\,430\,405\,208u^{37}$$
$$+226\,836\,378\,835\,893\tfrac{1}{19}u^{38}$$

$$L_{20}=2u^9+426u^{10}+12\,168u^{11}+116\,791u^{12}+408\,010u^{13}$$
$$-1\,725\,303u^{14}-28\,068\,380u^{15}-146\,244\,606\tfrac{1}{2}u^{16}+183\,286\,368u^{17}$$
$$+5\,831\,700\,720u^{18}+24\,903\,765\,412u^{19}-133\,290\,798\,124\tfrac{4}{5}u^{20}$$
$$-918\,229\,754\,186u^{21}+1\,010\,221\,776\,840u^{22}+33\,039\,627\,711\,312u^{23}$$
$$-65\,162\,985\,813\,622\tfrac{1}{4}u^{24}-536\,138\,254\,114\,502\tfrac{2}{5}u^{25}$$
$$+1\,197\,886\,310\,409\,025u^{26}+16\,769\,049\,034\,859\,628u^{27}$$
$$-135\,711\,480\,534\,609\,939u^{28}+539\,216\,938\,820\,525\,896u^{29}$$
$$-1\,427\,351\,587\,679\,843\,012u^{30}+2\,758\,701\,161\,989\,645\,172u^{31}$$
$$-4\,049\,513\,727\,032\,202\,413u^{32}+4\,593\,667\,664\,023\,828\,118u^{33}$$
$$-4\,044\,267\,354\,513\,888\,891u^{34}+2\,746\,573\,216\,492\,597\,121\tfrac{3}{5}u^{35}$$

$$-1\,414\,424\,109\,107\,792\,807u^{36}+534\,899\,462\,471\,120\,186u^{37}$$
$$-140\,258\,062\,342\,321\,550u^{38}+22\,801\,483\,748\,542\,438u^{39}$$
$$-1\,732\,269\,513\,128\,296\tfrac{7}{10}u^{40}$$

$$L_{21}=187u^{10}+7\,572u^{11}+98\,848u^{12}+508\,368u^{13}-72\,116u^{14}$$
$$-20\,885\,348u^{15}-154\,042\,028u^{16}-344\,920\,976u^{17}$$
$$+4\,007\,470\,414\tfrac{1}{3}u^{18}+31\,374\,518\,748u^{19}+25\,751\,748\,568u^{20}$$
$$-1\,093\,841\,166\,394\tfrac{2}{3}u^{21}-2\,626\,954\,396\,523u^{22}$$
$$+20\,337\,536\,503\,920u^{23}+129\,446\,442\,017\,216u^{24}$$
$$-724\,709\,432\,799\,592u^{25}-1\,510\,719\,967\,604\,758u^{26}$$
$$+12\,887\,433\,443\,079\,748u^{27}+42\,463\,121\,489\,599\,512\tfrac{6}{7}u^{28}$$
$$-669\,232\,628\,130\,268\,404u^{29}+3\,365\,914\,683\,950\,712\,007u^{30}$$
$$-10\,568\,722\,403\,619\,197\,532u^{31}+23\,761\,518\,696\,229\,231\,688u^{32}$$
$$-40\,399\,939\,863\,243\,421\,613\tfrac{1}{3}u^{33}+53\,284\,332\,867\,613\,477\,680u^{34}$$
$$-55\,095\,823\,925\,507\,382\,230\tfrac{2}{7}u^{35}+44\,682\,542\,766\,322\,989\,411\tfrac{1}{3}u^{36}$$
$$-28\,181\,444\,547\,798\,804\,700u^{37}+13\,566\,293\,816\,381\,457\,075u^{38}$$
$$-4\,821\,726\,325\,061\,722\,394\tfrac{2}{3}u^{39}+1\,193\,657\,285\,116\,677\,122u^{40}$$
$$-183\,919\,910\,093\,856\,064u^{41}+13\,288\,238\,510\,442\,588\tfrac{10}{21}u^{42}$$

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